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REVIEW OF SOME RECENT RESEARCH ON NOISE AND STRUCTURAL VIBRATION

by R. H. Lyon and G. Maidanik

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BOLT, BERANEK AND NEWMAN, INC.
Cambridge, Massachusetts

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REVIEW OF SOME RECENT RESEARCH ON
NOISE AND STRUCTURAL VIBRATION*

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Richard H. Lyon and Gideon Maidanik

SUMMARY

A lengthening roster of difficult questions concerning vibrations in very complicated structures--buildings, missiles, ocean vessels, etc.--has motivated a new approach to vibration analysis. These new techniques rely on an old trick: to make a "difficult" problem "easy," ask the easy questions. Often the answers to easy questions will suffice.

To exemplify these questions and some answers, a series of studies in the response of structures to sound and energy transfer between attached structures are discussed. For example, the response of ribbed panels to sound fields is analyzed. The procedure for computing radiation loss factor, which measures the coupling between sound and structure, and measurements of this parameter are described. Applications of the method are also made to estimate response of a large booster.

*This paper was originally prepared for oral presentation at a joint colloquium sponsored by the Department of Mechanical Engineering at the University of Kentucky and the Engineering Development Laboratories of the Electric Typewriter Division, IBM. For the present publication, the paper has been rewritten and some of the discussions have been expanded.

The method has also been applied to noise transmission through structural panels, sound radiation from a machine housing, and the transmission of vibrational energy from one structural element to another. In the last example, estimates of response variation as well as averages have been made. In all cases experimental studies have provided an important back up to the analytical development.

SYMBOLS

A_N	acoustic acceptance function
A_s	area of panel
c_ℓ	longitudinal velocity
c_o	ambient speed of sound
h	panel dimension in the y-direction
k_x	x-component of a wavevector in a panel
k_y	y-component of a wavevector in a panel
ℓ	panel dimension in the x-direction
M	mass of structure
m_s	mass per unit area
n_R	room modal density
n_s	structural modal density
S_a	acceleration spectral density
S_p	sound pressure spectral density
T_R	room reverberation time
T_s	structural reverberation time
t	panel thickness
V	room volume
v^2	mean square velocity defined by v^2M = structural energy
x,y	cartesian coordinates in the plane of a panel
η_{in}	internal loss factor
η_{rad}	radiation loss factor
ρ_o	ambient density
ρ_s	structural material density

SYMBOLS (cont.)

σ	radiation efficiency
ω_o	resonance radian frequency
ω	band center radian frequency
$\Delta\omega$	frequency bandwidth

I. Introduction

The problems of response of structures to acoustic noise and the subsequent vibration and reradiation of this noise are central to the field of acoustics. The applications of response studies are very broad, having important uses in the fields of missile response to environmental noise, aircraft response to sound and boundary layer turbulence, the vibration of buildings resulting from shock and sonic booms, transmission of noise through walls, etc. The missile and aircraft problems are primarily problems of vibratory response, the main problem areas being concerned with the fatigue of structures and environmental loads on equipment mounted within the structures. There is also the problem of acoustic noise transmission to the interior of aircraft cabins which are used for passenger and personnel accommodation. The analysis of the interaction of vibration and sound in buildings is primarily applied to the radiation of sound by the building walls. The walls may be excited by impacts or sound fields. The application of interest may be flanking transmission of vibration or the transmission of sound through the walls of the building.

The manner in which vibrating structures radiate noise is an important topic in noise control. It governs such effects as the radiation from machine housings, aircraft panels, submarine hull structures, building walls, etc. The parameter which governs the sound radiation is the radiation efficiency; it describes the power flow linkage between the structural vibration and the sound pressures which are transmitted to the environment. From a reciprocity argument, the radiation efficiency governs the response of structures to sound fields. The radiation efficiency of complex structures is discussed in this paper.

The transmission of noise through structures is a combined problem of response and radiation. The response may be either resonant or forced and the structure may be constructed from flat panels, as in the case of building walls and noise reducing boxes, or it may be formed by curved panels as in aircraft, missiles and submarines.

The dynamical equations for most of the systems mentioned above are reasonably well known and with the proper use of boundary conditions, one can formulate many of these problems in an exact manner. It is an unfortunate fact, however, that one is frequently interested in the response and radiation of structures in frequency ranges where very many modes of vibration are contributing to the response or where the appropriate wavelengths are small compared to the dimensions of the structures. This results in a very complicated pattern of vibration with very many degrees of freedom, a problem that not even a large scale digital computer is capable of handling with any ease. In addition, the interpretation of results of this type from a machine computation, or from an analytic computation, for that matter, can be extremely complicated when the vibration patterns are so complex. It is for this reason that an analysis which parallels the statistical theory of room acoustics (refs. 1,2,3) has been developed. In this analysis, the structure and its environment are described in terms of their average dimensions, modal densities, damping, etc., while the dynamical quantities of interest are energy, mean square velocity, mean square pressure, etc.

II. Statistical Theory of Structural Vibration

An example of the problems that concern us is shown in Fig. 1. A structure, which may consist of a combination of panels, frames, ribs, etc., is immersed in a reverberant sound field. This sound field is maintained by some distribution of sources within the space. It may be a rocket engine or electrodynamic speakers which are generating sound. The amount of energy which flows into the structure will depend on the degree of coupling between the structure and the sound field. One way of describing this coupling is through the radiation efficiency, σ, of the structure. Its value is determined by the thickness of the plates, the number of ribs, the geometry, etc.

The amount of energy which the structure will accept in any particular frequency band depends on how many modes will resonate in that frequency band and accept energy from the sound field. Therefore, an average modal density for the structure must be ascertained. This is achieved by considering the various elements which make up the structure and generate the total modal density by adding up the contributions of each part of the structure (refs. 4,5,6).

Once the energy has been accepted by the structure in its modes of vibration, it will either be dissipated internally or be reradiated back into the space. This loss of energy is expressed through a total damping which has contributions from radiation and from internal dissipation. The internal dissipation usually occurs at joinings of the structure (ref. 7) although it may be due to metal friction or hysteresis damping in some instances. Normally, however, hysteresis damping is of minor importance.

It is possible to approach the analytic analysis of the system in many ways, but perhaps some simple thermodynamic arguments will allow one to make some useful general statements about the response (ref. 8). Suppose the structure has only a single mode of vibration which resonates at a frequency within the audio spectrum. For this part of the spectrum, the air about the object is a continuum. If the continuum were at a temperature T, then the distribution of thermal energy would be governed by the Rayleigh-Jeans formula for the modes of a continuum in an enclosed space (the room). The spectrum would, accordingly, be well defined.

The resonant mode, however, only sees the energy of the modes of the room in the frequency range of its own resonance. If it has no damping, then the system is thermodynamic and the resonant mode comes to equilibrium with the modal energy of the modes which it sees in the room. Its modal energy is therefore fixed by the energy of the room and is independent of its geometry, or mass, or any of the details of its coupling to the room. The only parameter of importance is its resonant frequency and the fact that it is coupled, to some extent, to the motion of the room. For no damping, therefore, the energy that the structure achieves is finite and is equal to kT, the energy of the modes with which it comes to equilibrium (here k is the Boltzmann constant). When internal damping is added, then the amount of energy that the single mode attains is modified.

The modal energy of a damped resonant mode at equilibrium was derived by Smith (ref. 9). His formula is

$$Mv^2 = \frac{2\pi^2 c_o}{\rho_o \omega_o^2} S_p(\omega_o) \frac{\eta_{rad}}{\eta_{rad} + \eta_{in}} , \qquad (2.1)$$

where M = mass of the structure,

v^2 = mean square velocity of the structure, averaged in time and space such that Mv^2 = modal energy. We have called v the "kinetic velocity" of the structure,

c_o, ρ_o = ambient sound speed and density of the acoustic medium respectively,

ω_o = resonance radian frequency of the mode,

S_p = spectral density of the acoustic pressure,

η_{rad} = radiation loss factor. For a panel $\eta_{rad} = \rho_o c_o \sigma / \omega_o m_s$

m_s = mass per unit area,

σ = radiation efficiency; ratio of power radiated by a panel to that radiated by baffled piston of the same area and rms velocity at a frequency such that the wavelength is much smaller than a typical linear dimension,

η_{in} = internal dissipation loss factor of the structure.

That an increase in the radiation damping results in an increasing response is not paradoxial when one recalls that the radiation loss factor is merely a measure of the degree of coupling between the structure and the sound field. The stronger this coupling, the larger is the exchange of power between the sound field and the structure. When the internal dissipation in the structure vanishes or becomes small compared with the radiation damping, then there is an equivalence between the modal energies of the sound field and the structure. Thus Eq. (2.1) reduces correctly at the limit $\eta_{in} \longrightarrow 0$ (or $\eta_{rad} \gg \eta_{in}$) in accordance with the thermodynamical argument above.

Practical structures frequently contain several modes of vibration in the frequency bands (e.g. third octave bands) that measurements are usually made in. These modes interact with the modes of the acoustic field which lie in the same frequency band. This interaction is governed by an equation which is derived from Eq. (2.1) by multiplying it by the modal density of the structure,

$$\frac{S_a}{S_p} = \frac{2\pi^2 n_s c_o}{M \rho_o} \frac{\eta_{rad}}{\eta_{rad} + \eta_{in}} , \qquad (2.2)$$

where n_s = modal density of the structure in the frequency band characterized by the frequency ω,

$S_a = \omega^2 v^2/\Delta\omega$ = acceleration spectral density of the structural vibration; $\Delta\omega$ is the bandwidth.

In Eq. (2.2) η_{rad} and η_{in} are averaged over the frequency band. The purpose of setting the equation in terms of the spectral density and the modal density of the structure is that these quantities can be measured in most cases of practical interest. The modal density can usually be predicted. For a flat panel, Eq. (2.2) becomes

$$A_N \equiv \frac{m_s{}^2 S_a}{2S_p} = \frac{\pi\sqrt{3}}{2} \frac{c_o \rho_s}{c_\ell \rho_o} \frac{\eta_{rad}}{\eta_{rad} + \eta_{in}} \quad , \tag{2.3}$$

using

$$n_s = \frac{\sqrt{3} A_s}{2\pi c_\ell t} \quad , \tag{2.4}$$

where A_s = area of the panel,

$m_s = M/A_s = \rho_s t$,

t = thickness of the panel,

c_ℓ = longitudinal velocity of the panel material.

Eq. (2.3) is written such that a comparison with "mass law" response can be conveniently made. The mass law response is the non-resonant response of the panel to a randomly incident sound pressure field. In this case $A_N = 1$. The parameter A_N is termed the "acoustic acceptance function" of the structure and is usually plotted in decibels as a function of frequency.

III. Reciprocity Method of Deriving the Response Equation

Eq. (2.2) is central to the response prediction methods and therefore its physical interpretation must be clearly understood before one can proceed further to examine its application to practical cases. For this purpose it may be helpful to derive the formula by a simplified physical argument. The method of derivation is based on the acoustical reciprocity between two sources in a closed space.

Consider the system whose arrangement is shown schematically in Fig. 2. Shown in a closed space is a resonant panel structure which is driven by a point velocity source. The velocity at the driving point is denoted by v and is specified such that it has a constant spectral density over a bandwidth $\Delta\omega_o$ about ω_o. The velocity source produces a reverberant vibrational field on the panel whose amplitude is denoted by u. Over the frequency band the point input impedance of the panel, R, is resistive so that the power balance relationship for the panel may be written

$$\langle v^2 \rangle R = \langle u^2 \rangle M\omega(\eta_{rad} + \eta_{in}) \quad . \tag{3.1}$$

The left hand side of Eq. (3.1) accounts for the input power from the source and the right hand side accounts for the radiated power and the power which is dissipated internally in the panel. The radiated power is distributed in the enclosed space and results in a reverberant pressure field which is characterized by a mean square pressure $\langle p^2 \rangle$. Since $\langle p^2 \rangle$ is proportional to the radiated power one may write

$$\beta \langle p^2 \rangle = \langle u^2 \rangle M\omega\eta_{rad} \quad , \tag{3.2}$$

β being a factor of proportionality. If a small sphere is placed in the reverberant field, it will experience a pressure p on its surface.

Next the reciprocal case is considered. The small sphere is an acoustic velocity source which produces a volume velocity U which has the same spectrum as does v. The radiated power from this source produces a reverberant pressure field which is denoted by $\langle p'^2 \rangle$. Similar to Eq. (3.2) one may write for this case

$$\langle U^2 \rangle R^o{}_{rad} = \beta \langle p'^2 \rangle \quad , \tag{3.3}$$

where $R^o{}_{rad}$ is the radiation resistance of the acoustic velocity source such that the left hand side of Eq. (3.3) represents the acoustic power radiated by the source. Note that the constant of proportionality β in Eq. (3.3) is the same as that in Eq. (3.2). The reverberant pressure field $\langle p'^2 \rangle$ is felt on the surface of the panel and generates a certain reverberant velocity amplitude, u', on it. One may express the relation between p' and u' by

$$\langle u'^2 \rangle = \alpha \langle p'^2 \rangle \quad , \tag{3.4}$$

where α is a constant of proportionality.

It is essentially the parameter α which is desired in determining the responsiveness of the panel to a reverberant sound field. The blocked force at the velocity source, v, which we denote by f' is related to the reverberant velocity amplitude u' again by the point impedance of the panel R, thus

$$\langle f'^2 \rangle = R^2 \langle u'^2 \rangle \quad . \tag{3.5}$$

For pure tones there is a reciprocity relationship between the variables p, v, f' and U. This relationship is

$$\frac{p}{v} = \frac{f'}{U} \quad . \tag{3.6}$$

Since v and U were assumed to be white sources in the thought experiment, this reciprocity relation can be expressed in terms of the mean square of the variables;

$$\langle U^2\rangle/\langle f'^2\rangle = \langle v^2\rangle/\langle p^2\rangle \quad . \tag{3.7}$$

Substituting in Eq. (3.7) the expressions for the variables derived in the preceding equations one readily obtains the expression for the ratio

$$\alpha = \langle u'^2\rangle/\langle p'^2\rangle = (R\,R^o{}_{rad})^{-1}\,\frac{\eta_{rad}}{\eta_{rad} + \eta_{in}} \quad . \tag{3.8}$$

If R and $R^o{}_{rad}$ are expressed explicitly, then one finds that Eq. (3.8) is equivalent to Eq. (2.3) which was originally derived from the modal approach (ref. 8).

IV. Experimental Studies of Panel Response

A systematic study of the response of a ribbed panel, shown in Fig. 3, has been carried out. The panel was placed in a large room (30 x 20 x 16 ft.) and measurements of the response of the panel to reverberant sound fields, generated by a loudspeaker, were made. A series of measurements of the sound generated by a reverberant vibrational field on the panel were also made to directly evaluate η_{rad}. The acceleration spectral density of the panel and the pressure spectral density in the room are related by (ref. 8)

$$S_p/S_a = \omega M \rho_o T_R \eta_{rad}/27.6\pi^2 n_R c_o \quad , \tag{4.1}$$

where T_R = reverberation time of the room in the frequency band,

$n_R = \omega^2 V/2\pi^2 c_o^3$, = modal density of the room,

V = room volume.

Hence from measurements of S_p, S_a and T_R, when the panel is driven, the value of η_{rad} can be determined. Such measurements were made in third octave bands, the excitation being white noise from an electrodynamic shaker. The results are reported in Fig. 4 for several boundary conditions (unribbed, ribbed, with edges baffled and unbaffled).

Using Eq. (2.3) η_{rad} was determined indirectly from measurements of S_a, S_p and $\eta_{tot} = \eta_{rad} + \eta_{in}$ for the case where the room is driven by external sound source. The total damping η_{tot} was found from structural reverberation time measurements, the relation being

$$\eta_{rad} + \eta_{in} = (13.8/\omega T_s)[1 + (M_b v_b^2/M_p v_p^2)] \quad , \tag{4.2}$$

where T_s = reverberation time of the structure,

M_b, M_p = mass of the ribs and plate, respectively,

v_b^2, v_p^2 = mean square velocity of the ribs and plate, respectively.

The measurements were made in third octave bands, the drive spectrum was broad band noise. The results are shown in Fig. 4 for several boundary conditions (ribbed with baffled and unbaffled edges). It is noted from Fig. 4 that the agreement between the two ways of determining η_{rad} is satisfactory. This agreement may be interpreted as supporting the validity of the response equation, Eq. (2.2), which forms part of the procedure for obtaining the experimental points in Fig. 4.

From Eq. (2.2) it appears that if the response is to be predicted, one must be able to estimate the coupling factor $\eta_{rad}/(\eta_{rad} + \eta_{in})$. Thus, a method of estimating the quantities η_{rad} and η_{in} as functions of frequency must be sought.

Although some basic research has been performed on the estimation of the internal loss factor of built up structures (ref. 7), this research is not advanced sufficiently for making confident predictions at the present time. Therefore, one can only venture an educated guess at the appropriate values for this quantity or else look for measurements to obtain more reliable values. The usual ways of obtaining the experimental values of η_{in} are by measuring the reverberation time of the structure and using an equation such as Eq. (4.2) or studying modal bandwidths. Care should be taken that the vibrational field of the structure during the measurements is representative of the vibration that the structure is subjected to under operating conditions.

The quantity η_{rad} is important not only in response studies but also in the studies of noise generation by vibrating structures. In the following section a way of estimating η_{rad} is described.

V. Estimation of Radiation Loss Factor

In many noise control problems, the efficiency of the structure as a radiating body is very important because the radiation efficiency is the parameter which relates vibration to radiated sound. From the above discussion it is apparent that it also plays a governing role in the conversion of acoustic power into vibratory motion of structures. The parts of the structure which are usually most closely coupled to the sound field are the (flat or curved) panels of the structure since there is some possibility of trapping the air and compressing it as the panel vibrates. A simple form of panel vibration is the simple traveling wave such as shown in Fig. 5. Such a wave has a particular wavelength at any given frequency and this wavelength changes with frequency. Unlike a sound wave, the wave in the structure does not have a speed independent of the frequency. Rather, the speed increases with the square root of the frequency, as shown in Fig. 6. This figure shows the flexural wave speed for either aluminum or steel panels as a function of frequency for various plate thicknesses.

A particularly important parameter is the frequency at which a plate of any thickness has its bending wave speed equal to the sound speed. In air under standard conditions, this is about 1100 ft/sec. Note that for a 1" plate this is slightly over 500 cycles, for a 1/2" plate it is slightly over 1000 cycles; for 1/4", 2000 cycles; for 1/8", 4000 cycles, etc. At and above this frequency, which is termed the "critical frequency," there is a chance for the bending wave to match up very well with the acoustic wave giving a very large acoustic output from the panel.

Assuming that the acoustic loading does not affect the vibration of the panel, the radiation efficiency of such a wave versus the frequency to critical frequency ratio is shown in Fig. 7 (ref. 10). As one goes above the critical frequency, one notes that the radiation efficiency becomes nearly unity, i.e., the radiation impedance becomes that of the infinite wave impedance. Thus, it is concluded that when the frequency of the vibration of the panels of the structure is above the critical frequency f_c, there will be a large radiation output. Panel modes which resonate at frequencies above the critical frequency are called "acoustically fast." The simple theory of infinite plate flexural waves indicates no radiation below the critical frequency but one knows from experiment that this is not correct. The modes which resonate at frequencies below the critical frequency are called "acoustically slow."

To see how radiation is achieved for panels below the critical frequency, consider Fig. 8 (ref. 6). In this figure is diagramed one of the higher order modes of a simply supported panel. The mode shape consists of an alternate series of loops, positive and negative with respect to the neutral position. These loops are indicated as plus and minus cells of the panel. A simple way to consider the radiation from the panel is to consider the uncancelled volume velocity from the various cells. Thus, starting in the center of the panel, one appropriately pairs off the adjacent plus and minus pairs of cells, the volume velocity of these cells cancel except at the edges where there remains a half-cell width uncancelled by this procedure. A half-cell width corresponds to a quarter of a wavelength in the direction of the panel along which the cancellation occurs (the y axis in Fig. 8). This uncancelled edge half-cell acts like a standing wave radiator which may or may not have a wavelength larger than the acoustic wavelength. If its wavelength is larger than the acoustic wavelength, it acts as a strip radiator radiating sound somewhat less efficiently than a plate above the critical frequency but more efficiently than a simple point source. Modes for which this type of radiation can occur are called "strip modes" (ref. 6).

In Fig. 9 is shown the kind of radiation which occurs when both the components of wavelength in the panel are smaller than the acoustic wavelength. Then there is cancellation also along the x-dimension of the panel and one is left only with small "corner cells" radiating in the very extreme corners of the panels. These corner cells, being greater than an acoustic wavelength apart, tend to radiate independently, and because their sizes are small compared to the acoustic wavelength, they radiate like small point sources and have somewhat less efficiency than the strip modes discussed before. The panel modes for this type of radiation are called "piston modes" (ref. 6).

When the wavelength in a panel is small compared to an acoustic wavelength, there usually are several modes of vibration, some of which are piston modes and some of which are strip modes. By combining the number of these modes and their combined radiation efficiencies, one finds that the total effect is one of dominance of the strip mode radiation. Thus, one is led to the concept of the edges of the panels being responsible for the radiation below the critical frequency (refs. 6, 11). This edge radiation dominates until one gets to the frequency where the acoustic wavelength becomes larger than the panel dimensions and all the radiation is accounted for by piston modes alone, resulting in a less efficient radiation (ref. 6). Above the critical frequency the modes of the panel are such that no velocity cancellation occurs between adjacent cells, and a maximum radiation efficiency is obtained--namely unity. These modes have associated wave speeds which are above the wave speed of sound at the same frequency. Such modes are classified as acoustically fast.

The modal radiation efficiency for the acoustically slow modes, for both the strip modes and the piston modes, can be derived from the determination of the volume velocity of each cell and then calculating the radiation from the uncancelled cells (ref. 6). Due consideration must be given to the relative phases of the uncancelled cells if these are at distances less than about a third of the acoustic wavelength apart. In this case cells of the same phase combine to give twice the radiation efficiency. If they have opposite phases, dipole radiation results with correspondingly lower radiation efficiency. The modal radiation efficiency for the acoustically fast modes is essentially unity.

If one can determine the modal density as a function of frequency and classify the modes according to their modal radiation efficiencies, then the radiation efficiency of the structure can be computed as a function of frequency by adding the contribution of each mode and dividing by the number of modes. Using this procedure the radiation efficiency for flat panels (ref. 6) and cylindrical shells (ref. 12) has been derived and found to agree satisfactorily with the experimental data. Examples of the sort of agreement that one is able to obtain is illustrated in Figs. 4, 10 (ref. 13) and 11 (ref. 12). Note that in the case of a cylinder there is a peak in the radiation efficiency about the ring frequency, which is the frequency at which the longitudinal wavelength is equal to the circumference of the cylinder. The peak is caused by the curvature of the plate which tends to increase the rigidity of the panel with a consequent increase in the wave speed of the modes. Curvature effects are most pronounced in the vicinity of the ring frequency. In this region the effects are so strong that acoustically fast modes may occur. Because of their relatively large modal radiation efficiency, they dominate the radiative properties of the structure.

The radiation from flat panels and cylindrical shells in air is now sufficiently well understood to allow one to make fairly accurate estimates of the radiated sound power when the input vibration is known.

As an example of how radiation efficiency can cause the sound pressure radiation to differ markedly from the vibration spectrum, Fig. 12 is cited. This figure shows the vibration levels and sound pressure levels measured for a small rotating machine. In the figure, the velocity levels are in decibels re 1 cm/sec. The radiation efficiency is the estimated ratio of the radiation loss factor of the cover plate to a piston radiator at high frequencies. The critical frequency of the 1/6 in. cover plate is 3 kc. In the upper bands the agreement between the measured and the computed SPL is fairly good. A reduction of the radiated noise in this case might be achieved by using a thinner, more "limp" cover plate so that the critical frequency is moved to a higher frequency.

VI. Applications to Structural Response

The energy method has been used to predict the skin vibration of one of the large current missiles (ref. 14). Under static firing conditions, sound pressure and acceleration measurements were made, as shown in Fig. 13. The skin panels of the booster in the region where the measurements were taken are long and narrow with the longest side directed along the axis of the missile. This geometry of the panel causes an anomaly in the modal density (ref. 14) which is indicated in Fig. 14. Shown is the locus of modal resonances in the k_x, k_y space where k is the wave number $2\pi/\lambda$. Note that as the constant frequency line, which is a quarter of a circle in this plot, intersects the cross resonance frequencies of the panel, a large group of modes occur almost simultaneously. It also happens that the group of modes which come in at this time are most strongly coupled to the sound field (strip modes). Thus, one may expect strong interaction between the panel and the sound field at these cross resonance frequencies. In Fig. 14, the second cross mode frequency $f = f_2$ is depicted as an example.

In Fig. 15 are shown the measured normalized response curves for the panel. These were measured with Stage I engine fired and Stage II engines fired respectively (ref. 14). Note that generally the response in the region of frequencies from 250 cycles to 1000 cycles is 20-25 db above mass law response (mass law governs the response of limp structures, see Eq. 2.3). Assuming an edge absorption of 1%, the theoretically predicted response is shown as a dashed line for a purely reverberant sound field and as a solid line for a traveling wave purely axial with the cylinder. The effect of the strong coupling at the cross modes is clearly evident from the predicted response and it is somewhat supported by the nature of the measured response curves. One does not know what the actual damping in the structure was since this quantity was not measured for the missile. The estimates, therefore, must have some uncertainty in terms of absolute magnitude.

In the calculations cited above, curvature effects have not been considered. The ring frequency for the tanks on Stage I is about 550 cps. Since curvature effects tend to raise the radiation efficiency above that of a flat panel at frequencies below and about the ring frequency (see Fig. 11), a peak in the acceptance function in this frequency region is to be expected. Whether the observed peak can be substantially accounted for by curvature has not been determined. Nevertheless, there is sufficient encouragement from these early estimates to plan further work on this general approach in predicting response levels.

VII. Applications to Noise Transmission

The transfer of acoustic energy through a panel which separates two acoustic spaces is, of course, a very old problem in the theory and technology of acoustics. Most calculations have assumed that it was the forced wave or mass law response of the panels which was primarily responsible for the noise transmission (ref. 15). In the discussions in connection with the response of a missile skin to acoustic excitation, it became clear that the vibration may be well above the mass law levels. This may also be true in the structural wall problem although in some frequency region the mass law calculations of transmission loss may not be too far wrong. This is because the mass law motions, being ipso facto of wavelengths greater than or equal to the acoustic wavelength, will radiate with greater efficiency at lower frequencies than the reverberant multimodal vibration (refs. 6, 16).

Some calculations of the average noise transmission through panels have been made using the method which is described in the preceding sections (ref. 16). It is of particular interest to compare these estimates with the "classical" forced wave transmission in the frequency regimes where the two would be of equal pertinence. The motivation for the study is the attempt to isolate noise from delicate electronic packages by enclosing them in a small box. These boxes are frequently much smaller than an acoustic wavelength at the lower frequencies. It became apparent that the traditional methods of calculating noise reduction through panels would not be applicable in this frequency regime. A diagram of the system under consideration is shown in Fig. 16.

The noise reduction is computed separately in three frequency regimes. At very low frequencies, both the panel and the interior volume of the box are stiffness controlled. The noise reduction of the box is frequency independent in this regime and achieves a level dependent upon the thickness and the stiffness of the panel and the dimensions of the enclosed volume. This simple behavior occurs at frequencies which are below the first panel resonance. In the second intermediate frequency regime, the panel has resonant behavior but the interior volume remains stiffness controlled. This persists until the first acoustic resonance of the enclosed space is reached. At this point, one enters the third, or high frequency regime, where the interior space becomes resonant and very soon reaches a high frequency asymptotic multimodal behavior.

The theoretical noise reduction for a 6" x 12" x 1/16" aluminum panel which forms a flexible side for a box of dimensions 6" x 6" x 12" is shown in Fig. 17. Here one can see the constant noise reduction predicted at low frequencies, the fluctuating noise reduction predicted in the frequencies between the first panel resonance and the first acoustic resonance, and then finally the high frequency behavior. In the mid-frequency regime, an average was taken over many modes of the panel response and then an average assuming that only one mode of the panel is vibrating at any time. These should give the average noise reduction and the minimal noise reduction in this frequency range respectively. Note that in the high frequency regime, the estimates of the free wave noise reduction lies below that of the forced wave. This implies that for this panel it would be incorrect to calculate the noise reduction based on a forced wave response. Since the free wave response is sensitive to damping, the damping would be important in this instance in determining the noise reduction.

Some measurements have been made on a system similar to the one discussed above (ref. 17). The experimental setup is shown in Fig. 18. In this experiment, an aluminum box was hung in a reverberant sound field. The noise reduction was determined from measurements of the external and internal sound pressure levels and is shown in Fig. 19. The first panel resonance for the box occurred at about 250 cycles and the first acoustic resonance of the interior of the box at about 630 cycles. Note that at frequencies below about 160 cycles the noise reduction is essentially independent of frequency as predicted. Between 250 and 630 cycles, the lower limit minimal reduction "single mode" curve which is computed for the panel and the average noise reduction are shown. In a general way, these define the mean and lower limits of noise reduction in this frequency range. A better agreement between theory and experiment could be hoped for. In the multimodal acoustic region, above the first acoustic resonance, from 1 kc to 6 kc, there is a fair degree of agreement between the noise reduction observed for the panel and the theoretical calculations based on a resonant panel analysis. In spite of the rather complicated behavior in the three regimes, the statistical approach has given a fair degree of insight into the observed noise transmission in a small box.

VIII. Applications to the Vibration of Connected Structures

The statistical theory of structural vibration can be extended to problems other than the interaction between sound fields and structures. One important area is the equipment environmental problem in missile and aircraft structures. In these structures, one has a set of skin panels which have accepted energy from the environment and are engaged in random vibration. To these panels are attached frames and stringers and to the frames may be attached equipment mounting trusses or other internal panels. The amount of energy which is transferred to the internal structure is important in establishing the environment of any equipment which may be mounted on that structure. Thus, the study of the transmission and sharing of energy by connected structures has an important bearing on understanding the total environmental picture. As a start, two cases of connected structures were studied; the first case consists of a beam cantilevered to a flat panel (refs. 18, 19), and the second consists of two panels which are joined at right angles to each other (ref. 19).

In Fig. 20 is shown a large, flat panel which has an aluminum bar cantilevered to it. In some of the experiments, a damping strip was applied, as shown, to the beam to control its internal damping. It is presumed that the transfer of moment and angular velocity at that junction is responsible for the energy transfer. The coupling parameter is therefore related to the moment impedances of the beam and the plate. These determine a coupling loss factor analogous to the radiation loss factor of section II.

The large panel was driven at the indicated driving point by a shaker excited in either 8% or octave bands of random noise. The cantilevered beam had modes of vibration which could be determined from resonance experiments. In part of the experiments, an 8% filter was used in the shaker circuit to probe frequency regions where large transfer of power from the plate to one of the beam modes would result. In these instances, the beam response was essentially limited to that of a single resonator. The energy of the beam was sampled at the free end since all the modes have an antinode of motion at that point. In addition, the plate was excited in octave bands of noise. These bands normally may encompass three or more modes of vibration of the beam which are excited well by interaction with the plate. Theoretical calculations of energy transfer according to a relation like (2.3) are used to determine the response. These involve the point moment impedances for the plate and the beam and the beam damping.

We shall not detail these calculations here, but basically they are quite similar to those which were presented for the acoustic interaction problem. Since in any frequency band the number of modes in the substructure--the beam--is small, there is the possibility of considerable fluctuations in the response from the theoretical average. These fluctuations occur because the beam is sensing the vibration of the plate at only one point, and therefore the coupling between some modes may be enhanced and in others suppressed. These variations may be computed and put into the form of a "safety factor" which must be added to the average response for one to have a certain confidence level that the observed response of the beam will lie below this level. This form of presentation would have some advantage to a designer whose concern was the avoidance of malfunction or fatigue of the substructure.

In Fig. 21 are plotted the results of the calculations (ref. 19). The line labeled "theoretical broadband" is the theoretical response ratio of beam to plate for a large number of interacting modes. Note that the theoretical values of the beam response normally lie above the plate response since the decibel value is positive. The octave band response ratios are indicated by the open circles on the diagram. The departure from the theoretical value is no more than about 1.5 db and this occurs in a region where only one or two modes are being excited in the octave band.

For the 8% band excitation, there are considerably more fluctuations in the experimental data which is indicated by the open triangles. These data are to be compared with the average "theoretical single mode at resonance" curve. The computed safety factor in the response to achieve 80% confidence for this example is 2.5 db. If this is added to the average estimate and the resulting estimation is called the 80% "confidence level", it is noted that all of the triangular ratios lie below the theoretical value. Of course, there is no certainty that this would occur, since an 80% confidence is not very high and it would be quite possible for one or more of the triangles to lie above the 80% confidence level curve.

In Fig. 22 is shown a similar experiment which was run for two plates welded together (ref. 19). The ratio of thicknesses of plate 1 to plate 2 is 1:2. For this ratio, the maximum value of the response ratio is 6 db; plate 1 vibrating 6 db more than plate 2. Note that the theoretical curve gets close to this (about 5 db) in some of the frequency ranges. The measured response ratios are generally within 2 to 3 db of the theoretical values even though not all of the theoretical and experimental values were measured in the same third octave bands. There might have been a better agreement if the same bands were used. Nevertheless, one probably would be satisfied with these theoretical predictions as an engineering estimate for most environmental problems.

IX. Concluding Remarks

In the course of this paper, we have tried to outline very briefly the broad nature of the research which has been carried out on the statistical description of structures and their response. The work was done chiefly under sponsorship of the NASA, the Air Force, and the Navy. This research is still in its early stages, in particular with reference to its applications and to establishing confidence in the methods and an understanding of their limitations. The analysis of variance which was discussed with respect to connected structures is one such example. In this case, some of the features of the statistical averages were unsatisfactory and the method was extended to the computation of safety factors for the predicted quantities. An approach of this type may some day form a solid part of the noise control technology. At this time, however, it remains a reference point for many measurements that cannot be explained or computed in other ways.

We should like to thank many of our colleagues at BBN for many stimulating discussions and their contributions over the course of this research effort.

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10. If the acoustic loading on the plate is taken into account, the singular peak in the radiation efficiency at the critical frequency is absent. The large radiation efficiency at or about the critical frequency makes the radiation damping so large that one cannot ignore its effect on the vibrational field of the plate. It turns out that the balance between these two effects tends to eliminate the peak.

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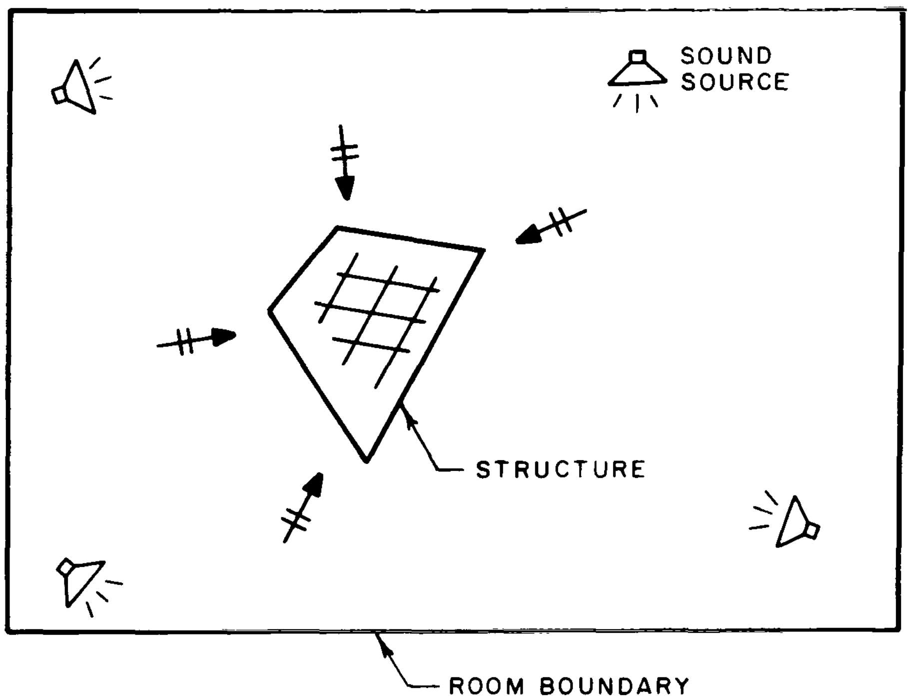

FIG. I PANEL-FRAME STRUCTURE IN A REVERBERANT SOUND FIELD

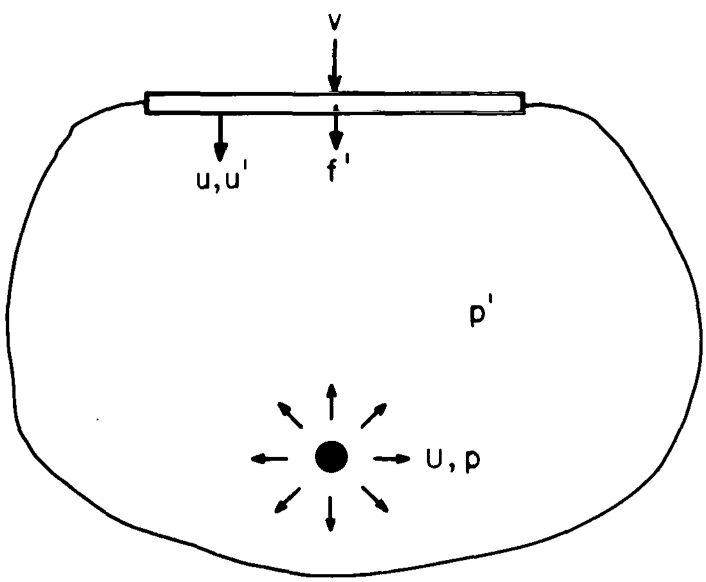

FIG. 2 RECIPROCITY EXPERIMENT WITH RESONANT PANEL AND ACOUSTIC MONOPOLE

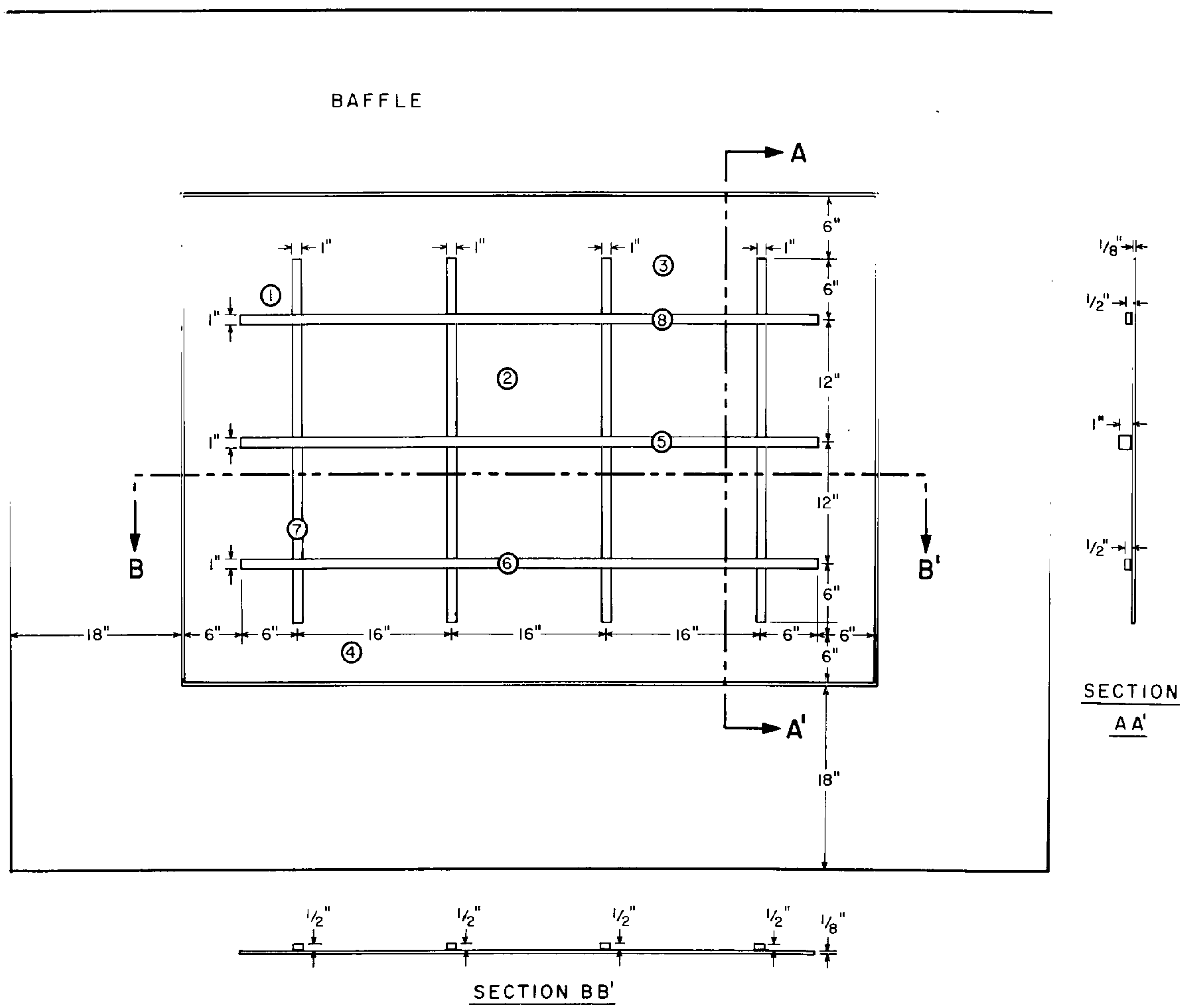

FIG. 3 RIBBED PANEL USED IN EXPERIMENTS OF STRUCTURAL RADIATION AND RESPONSE (REF. 6)

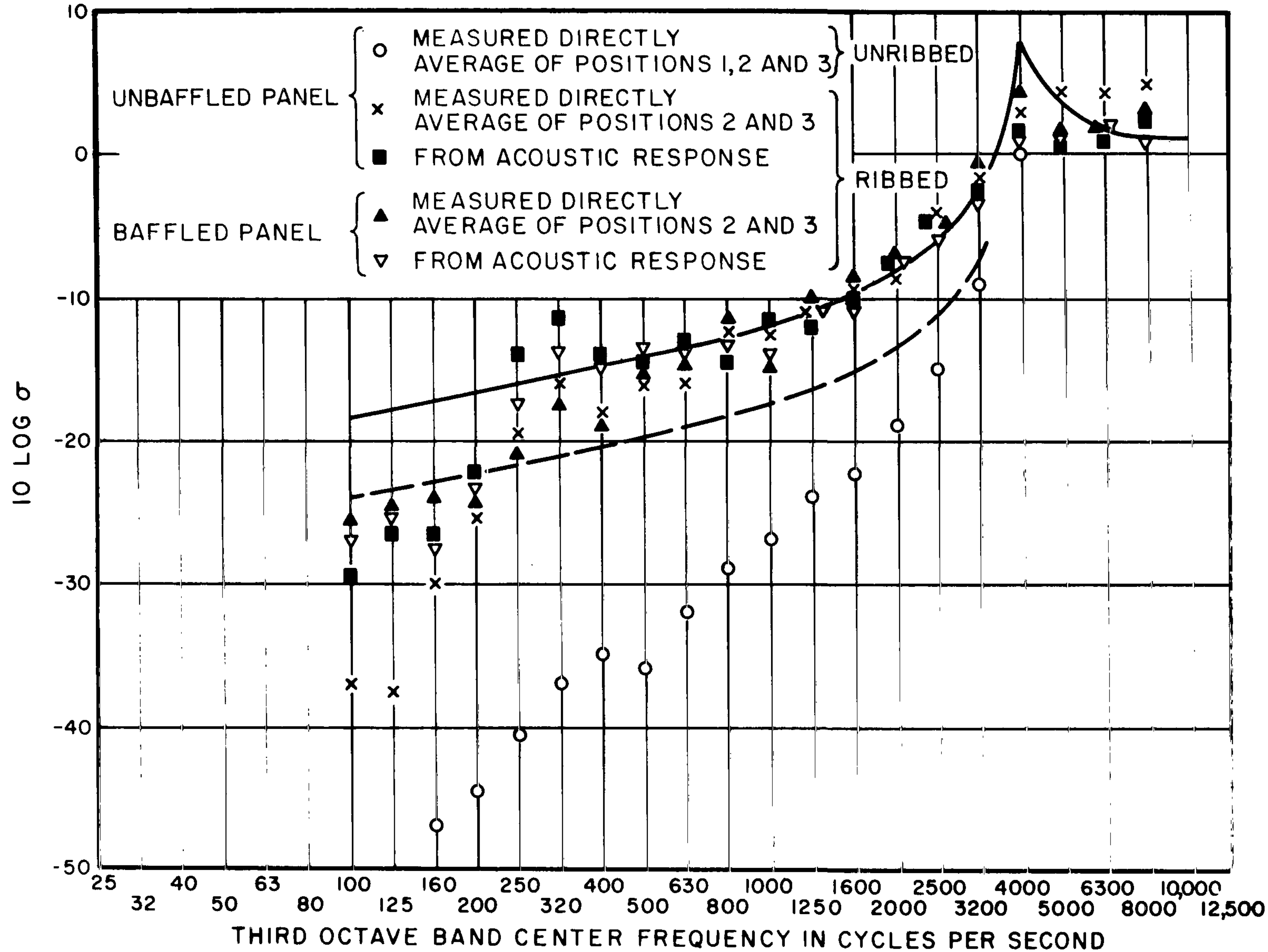

FIG. 4 RADIATION EFFICIENCY OF RIBBED PANEL IN FIG. 3 (REF. 6)

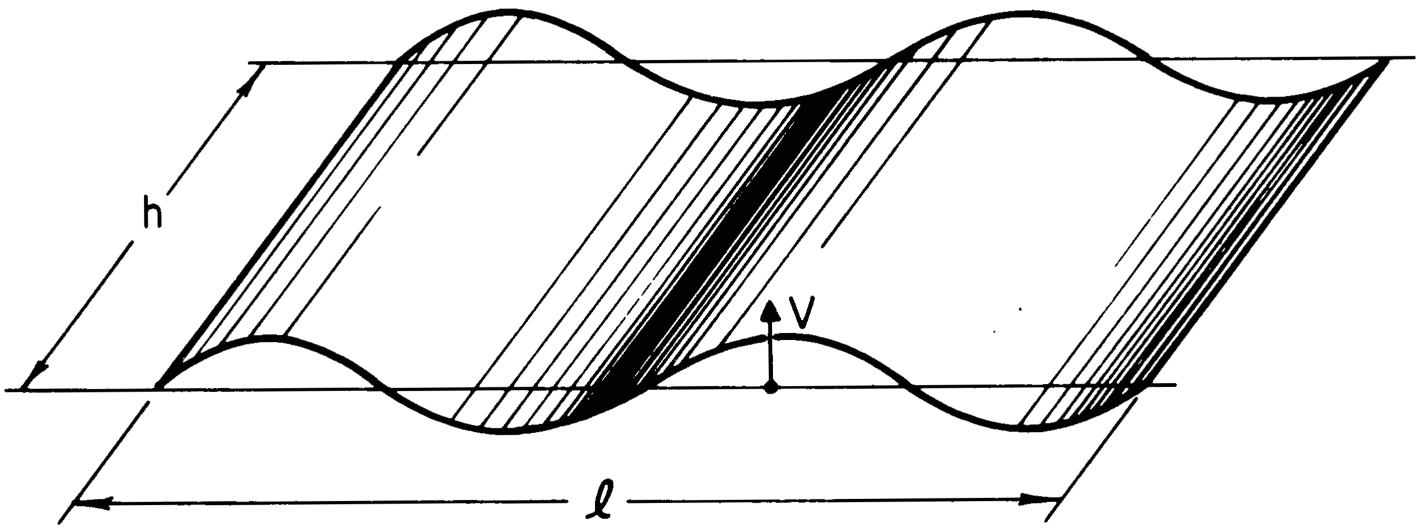

FIG. 5 FLEXURAL WAVE ON A THIN PLATE

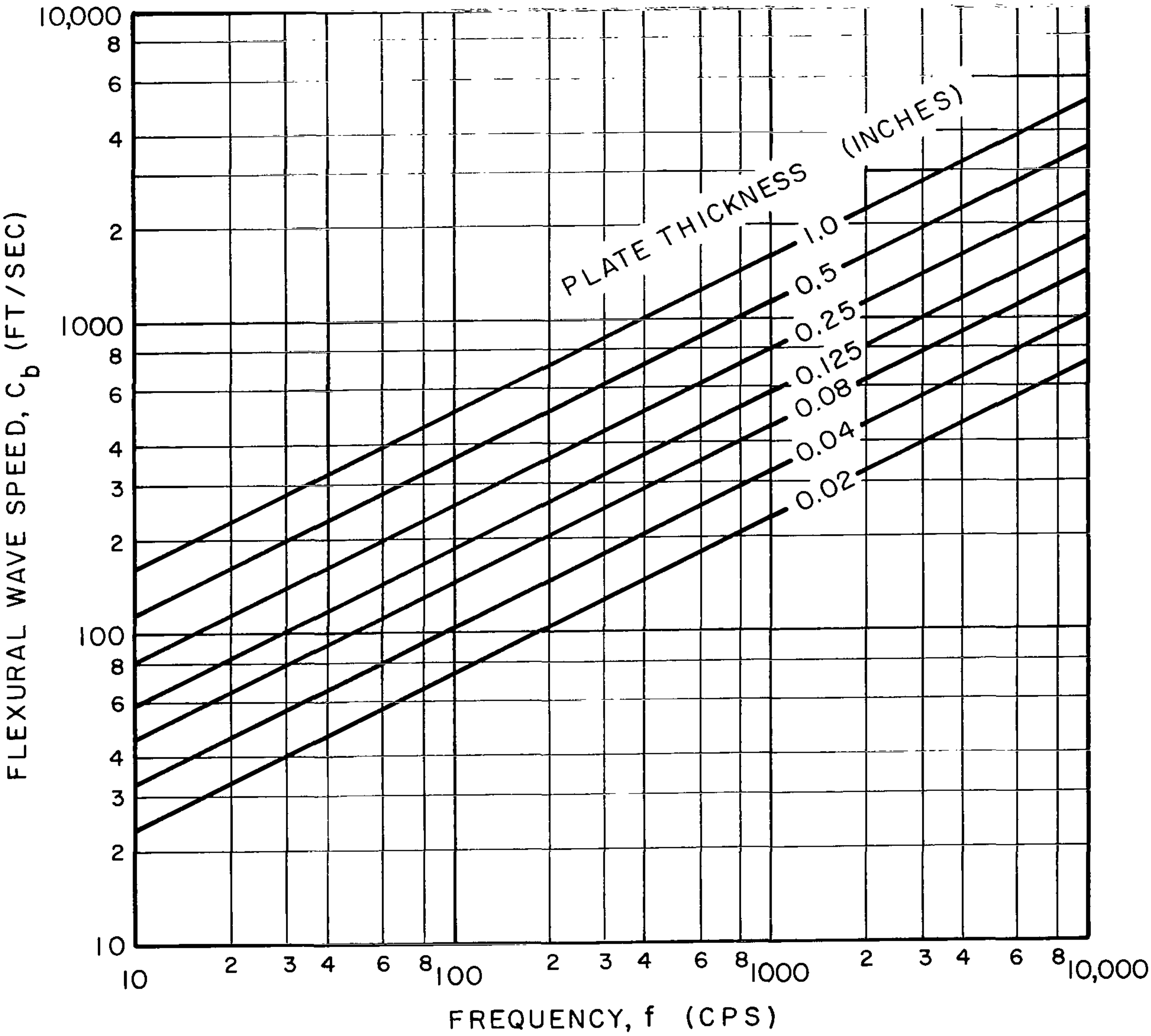

FIG. 6 FLEXURAL WAVESPEED AS A FUNCTION OF FREQUENCY FOR UNLOADED STEEL AND ALUMINUM PLATES

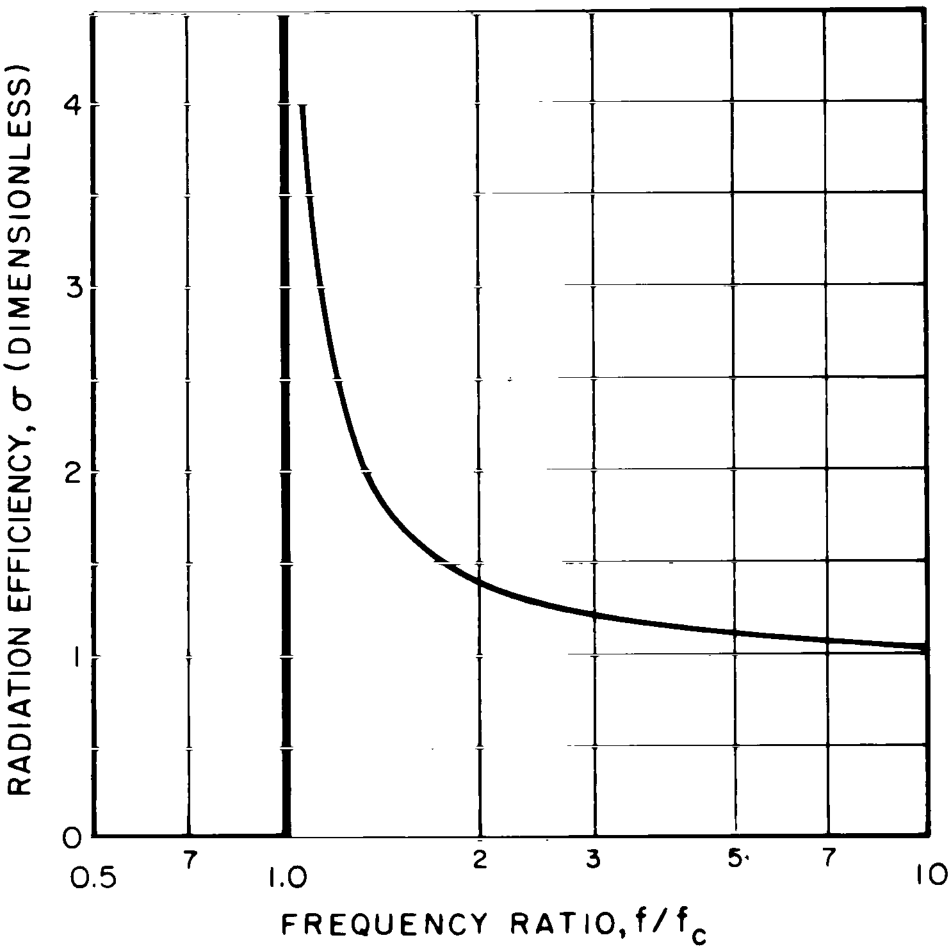

FIG. 7 RADIATION EFFICIENCY OF AN INFINITE PLATE

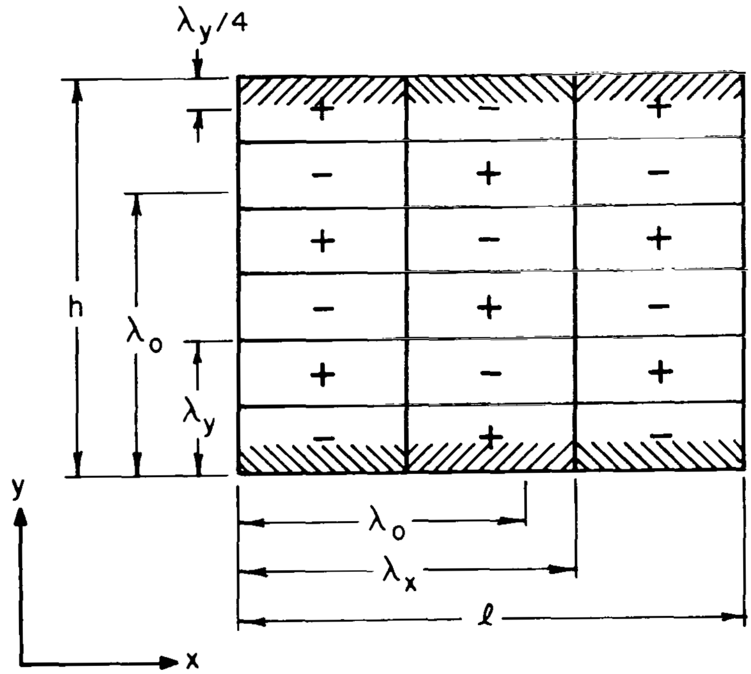

FIG. 8 VOLUME VELOCITY CANCELLATION FOR "STRIP MODES" (REF. 6)

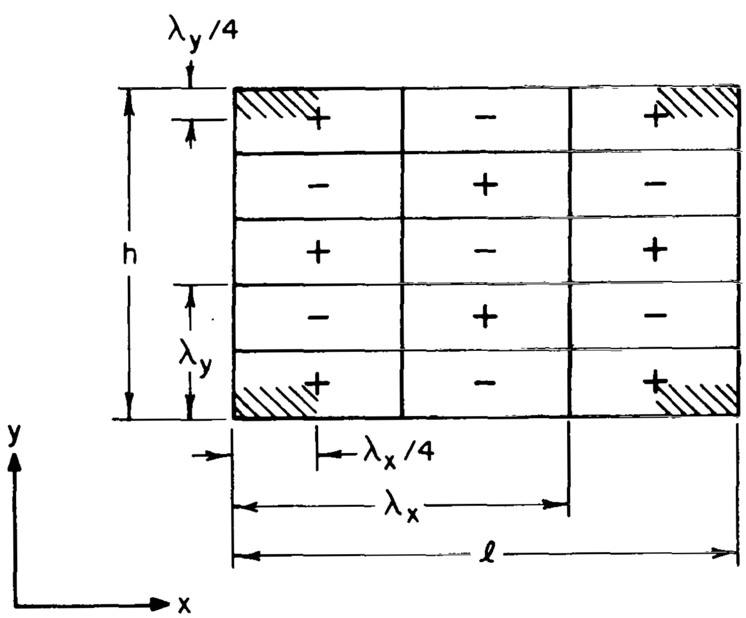

FIG. 9 VOLUME VELOCITY CANCELLATION FOR "PISTON MODES" (REF. 6)

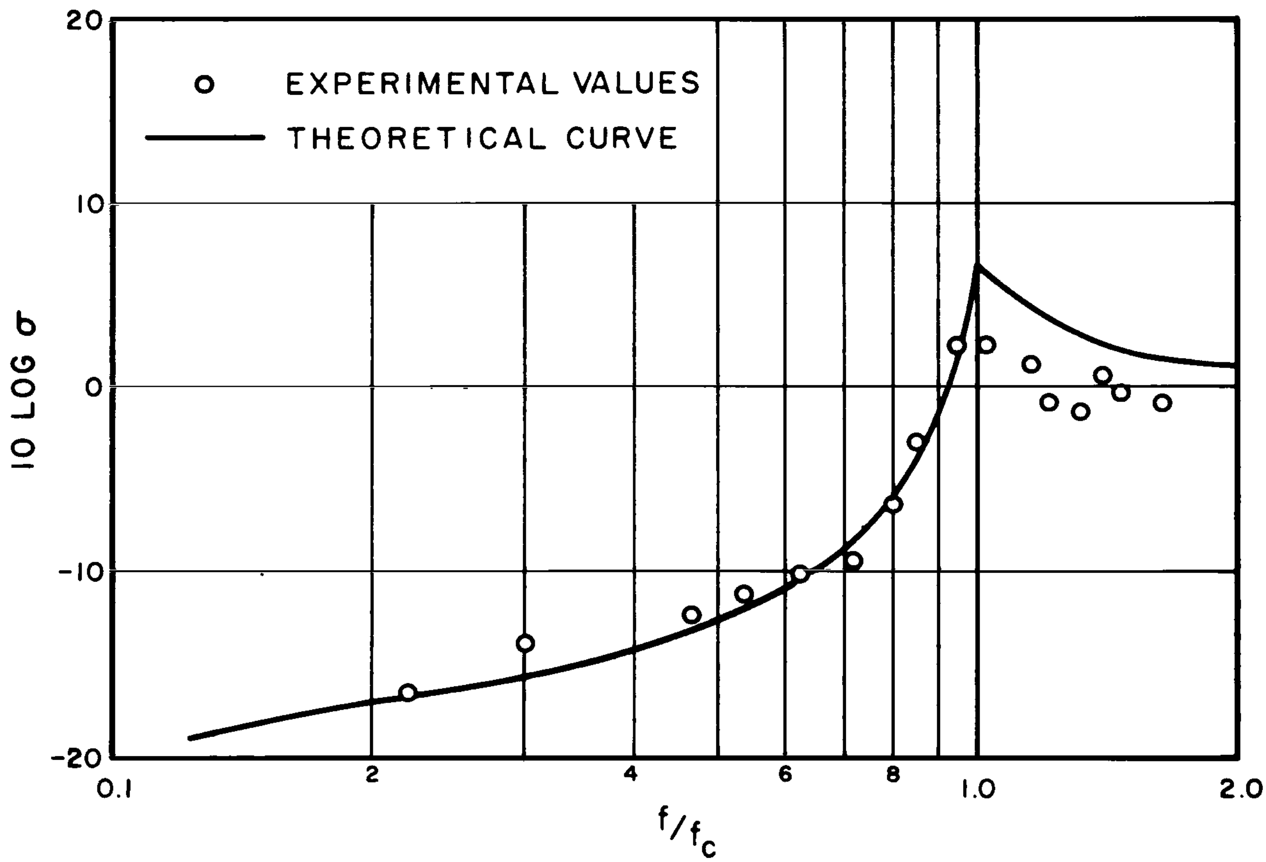

FIG. 10 RADIATION EFFICIENCY OF A BAFFLED PANEL (REF. 13)

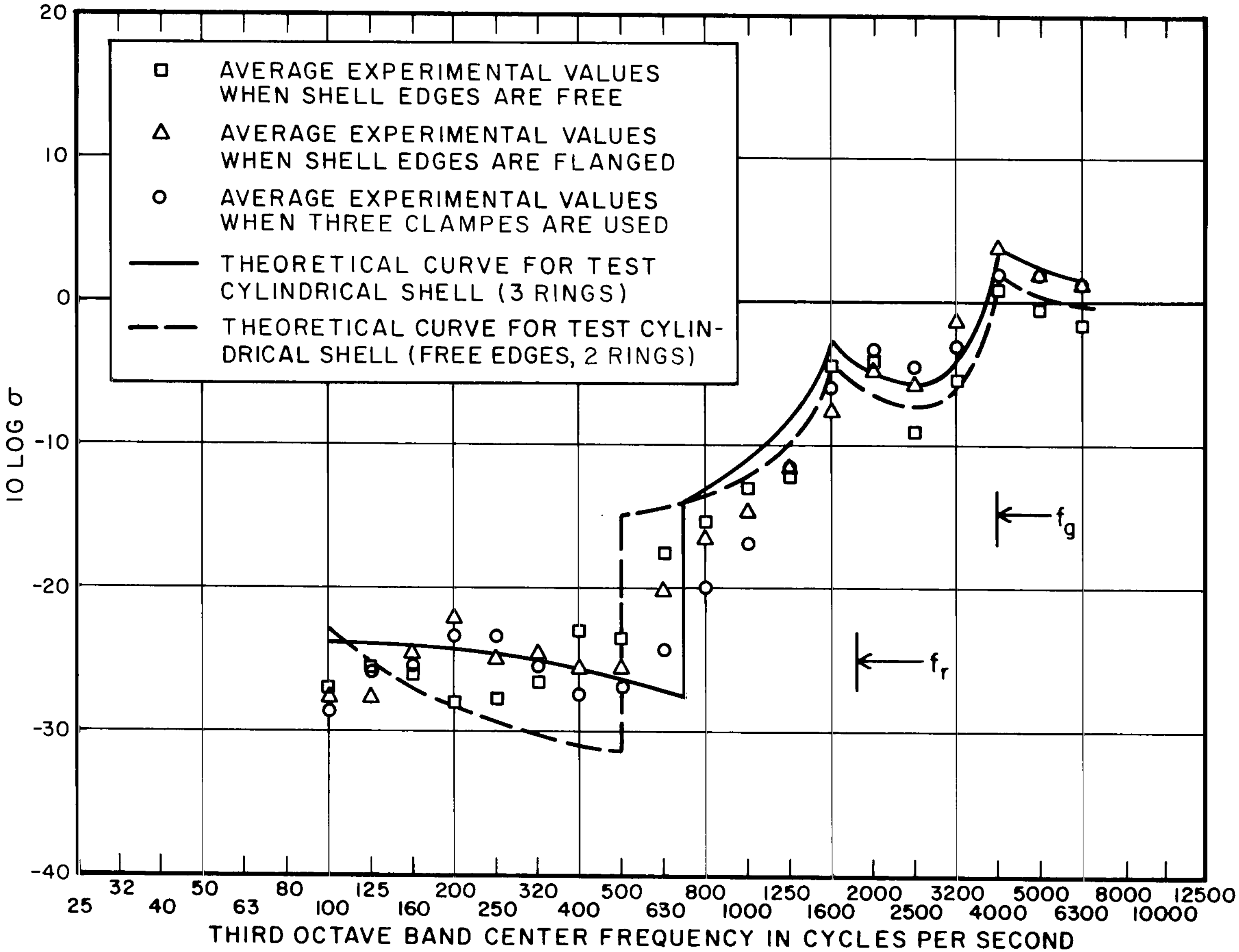

FIG. 11 RADIATION EFFICIENCY OF CYLINDRICAL SHELL (REF. 12)

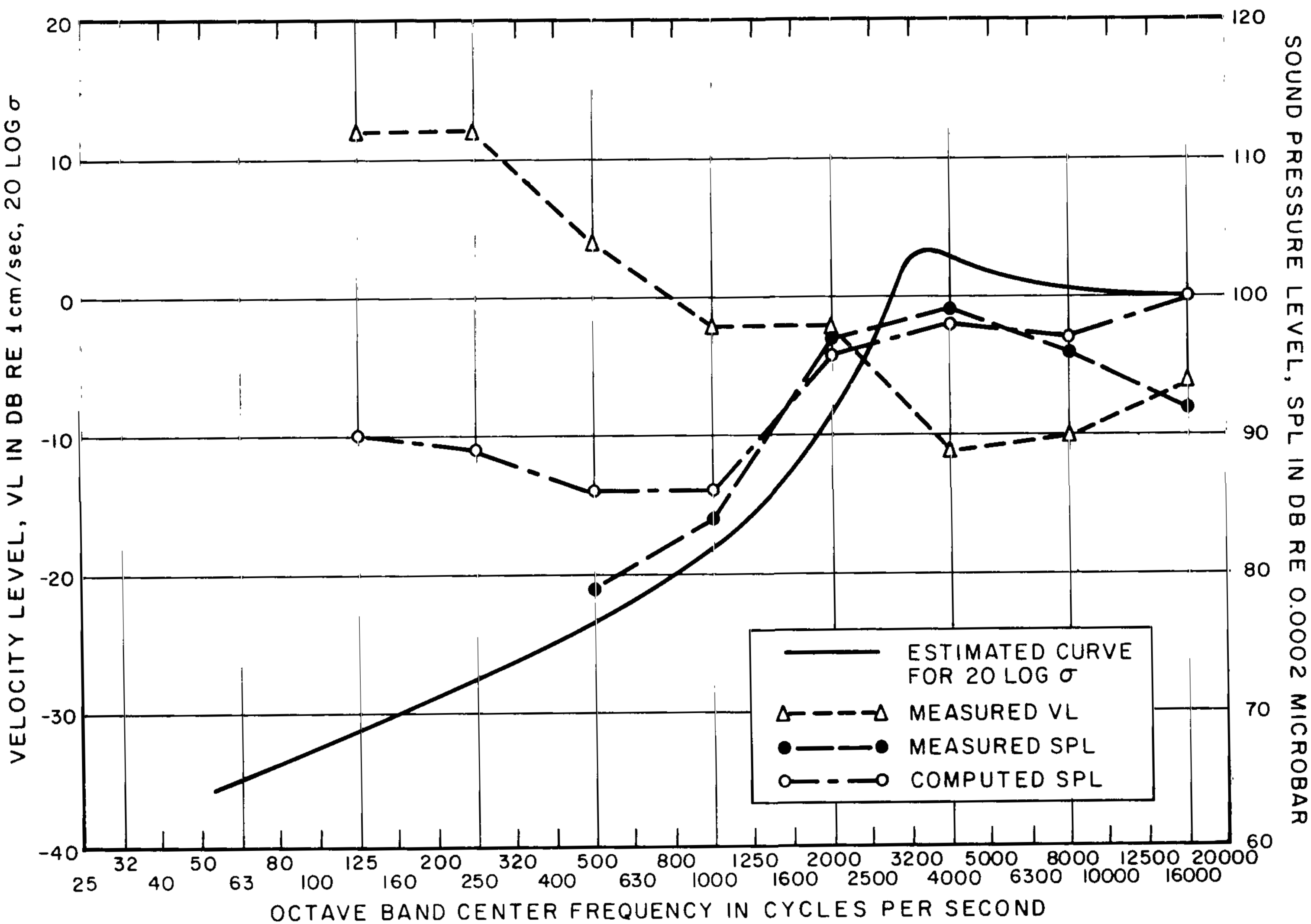

FIG. 12 RADIATION FROM MACHINE HOUSING

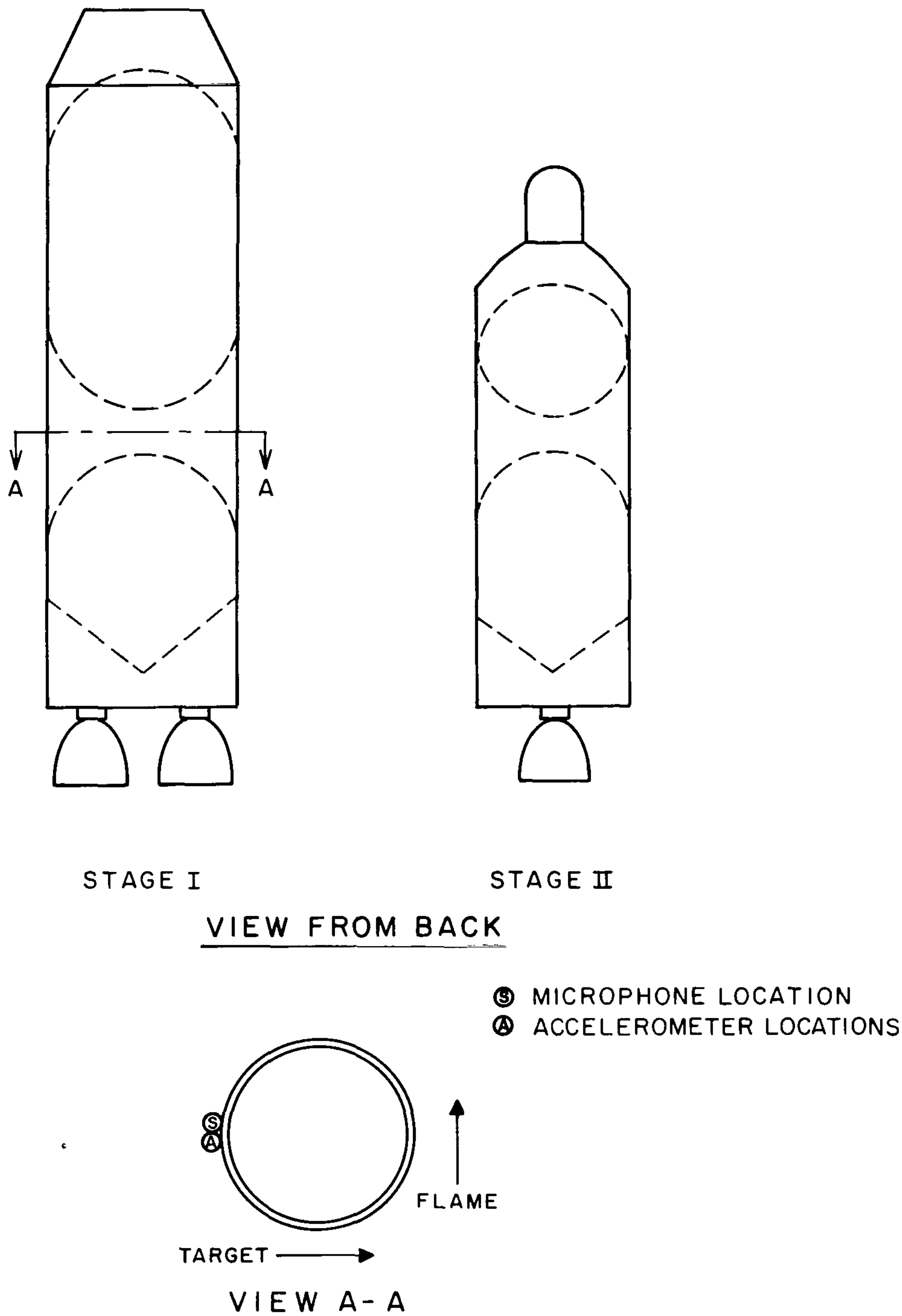

FIG. 13 LOCATIONS OF ACOUSTIC AND VIBRATION TRANSDUCERS ON BOOSTER (REF. 14)

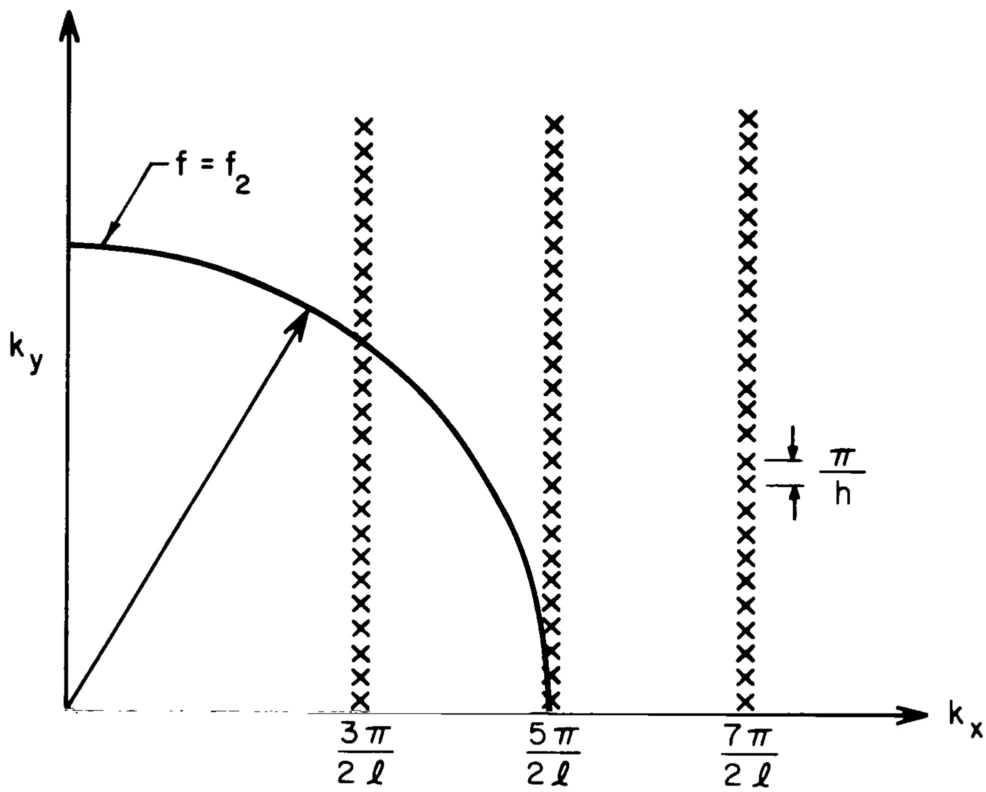

FIG. 14 MODAL PATTERN IN WAVE NUMBER SPACE FOR A PANEL WITH LARGE ASPECT RATIO (REF. 14)

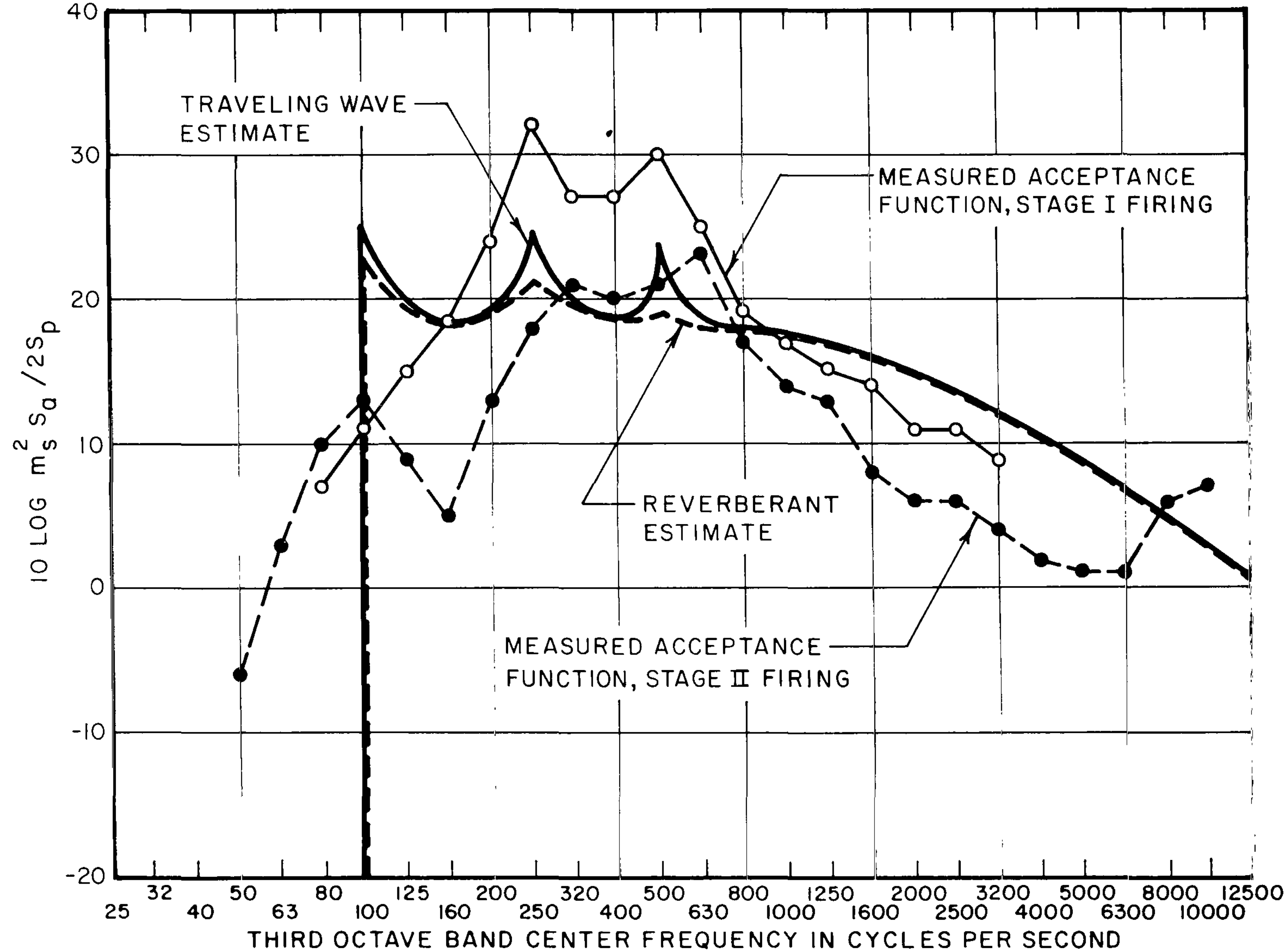

FIG. 15 COMPARISON OF ESTIMATED AND MEASURED ACCEPTANCE FUNCTIONS FOR BOOSTER $\eta_{in} = 10^{-2}$ (REF. 14)

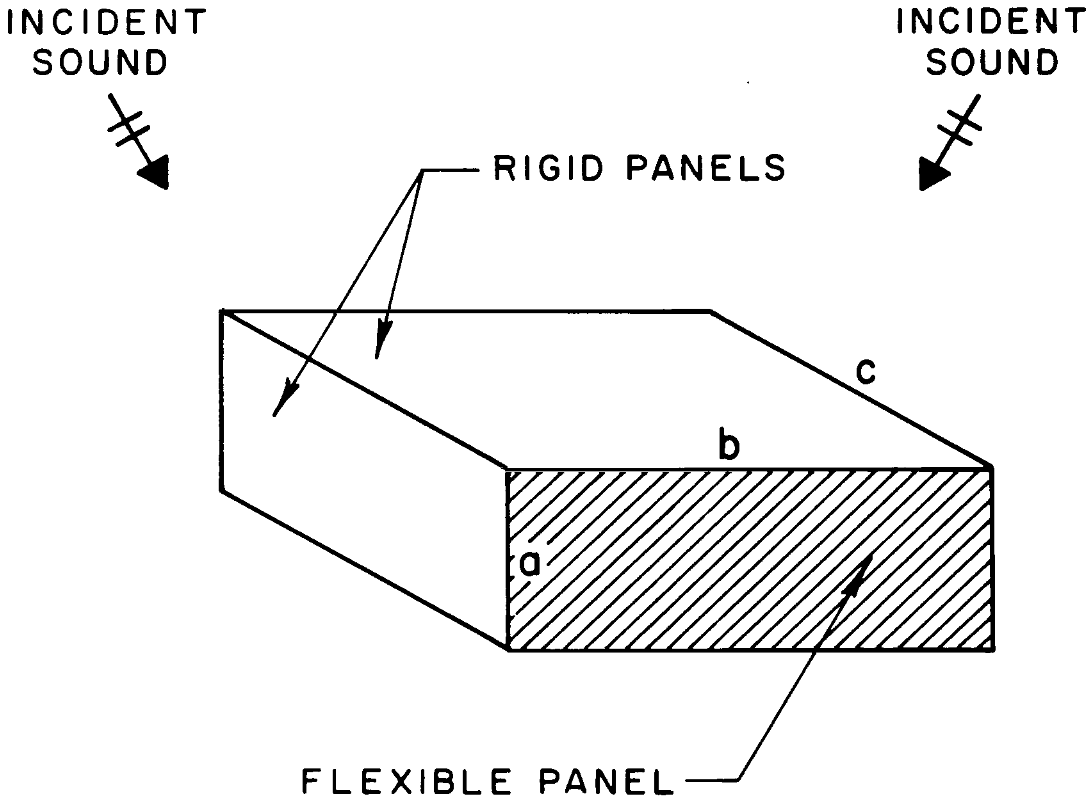

FIG. 16 RIGID ENCLOSURE WITH A SINGLE FLEXIBLE FACE SUBJECTED TO ACOUSTIC EXCITATION (REF. 16)

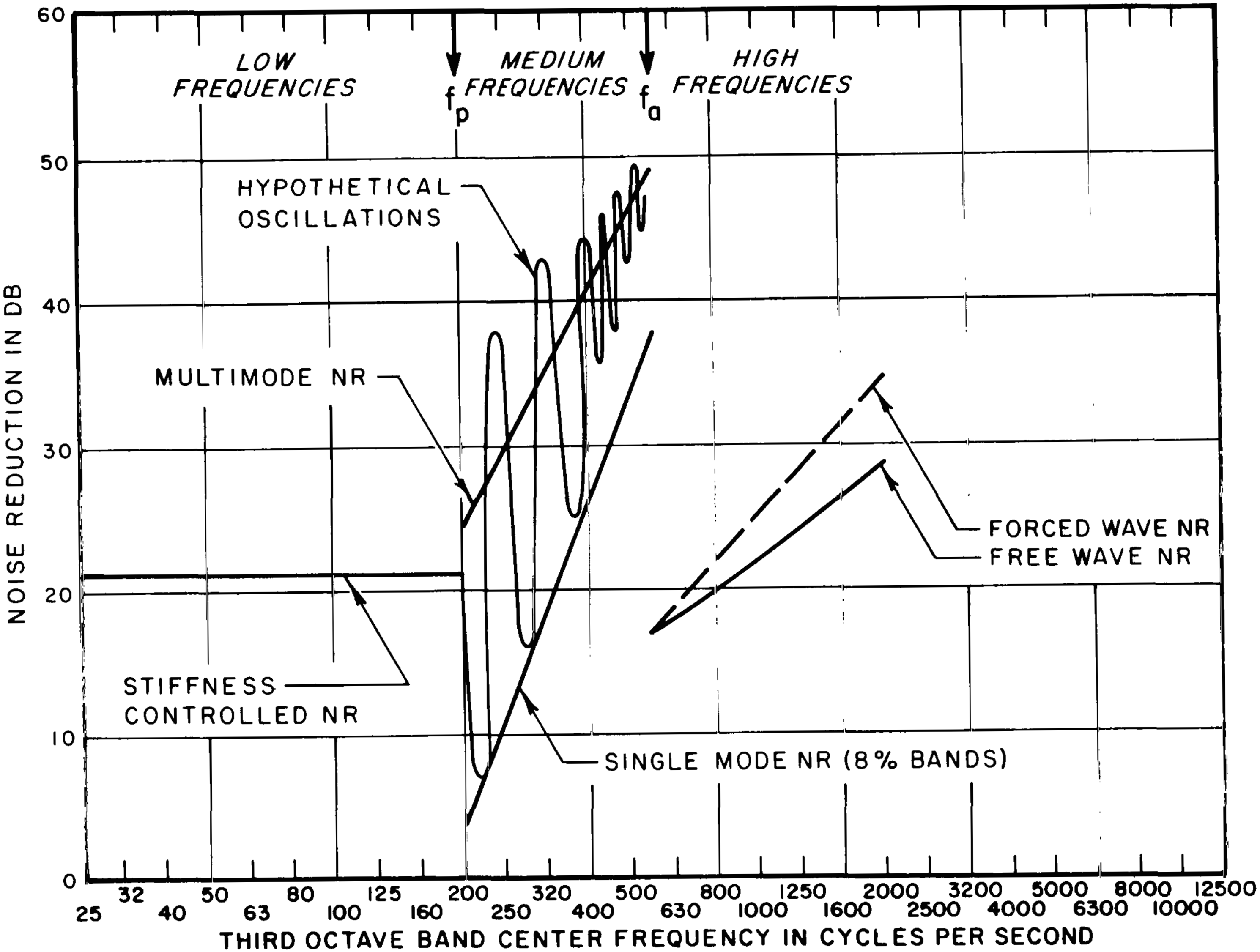

FIG. 17 THEORETICAL NOISE REDUCTION OF A SINGLE ALUMINUM PANEL IN RIGID ENCLOSURE (REF. 16)

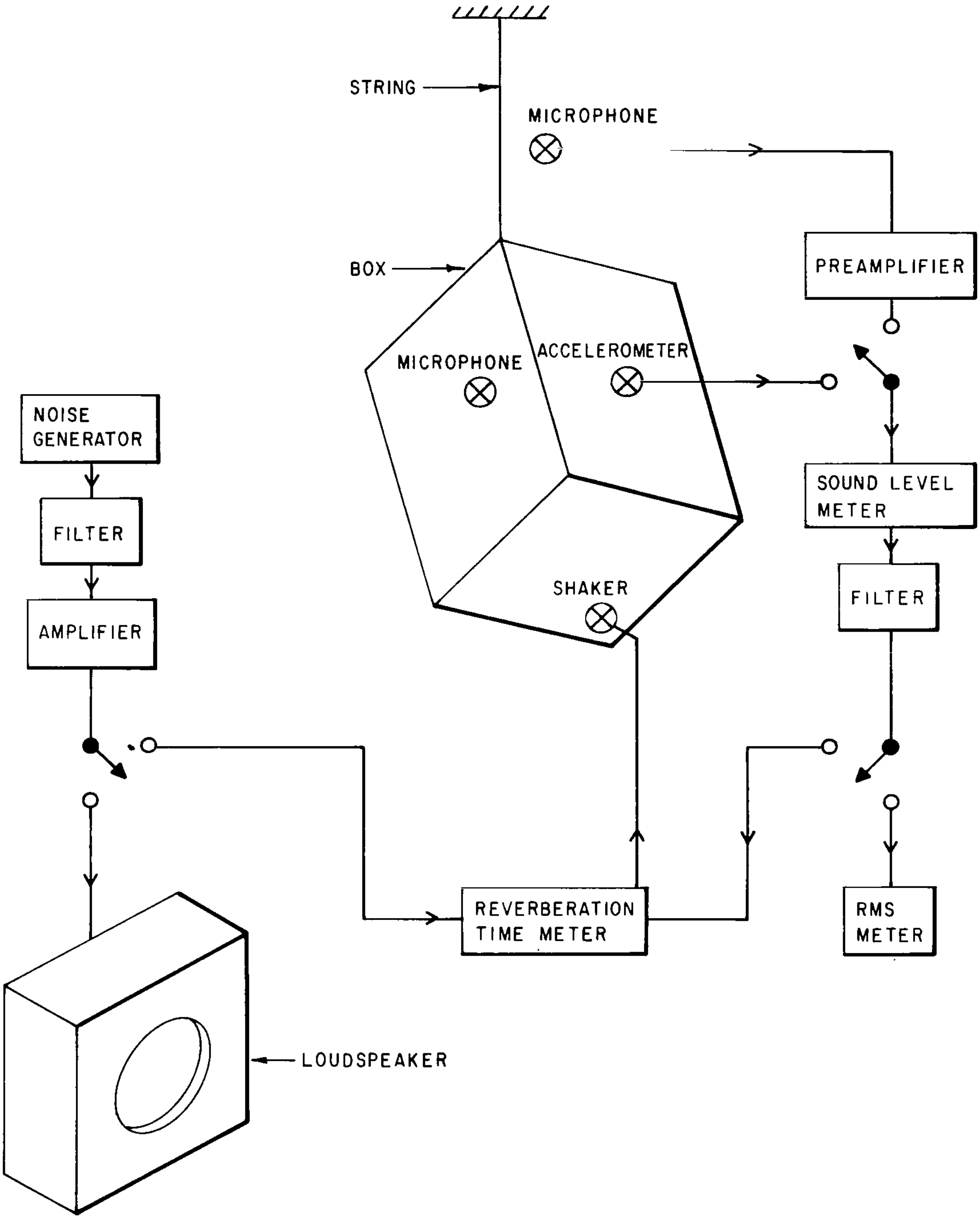

FIG. 18 EXPERIMENTAL SET-UP FOR NOISE REDUCTION STUDIES OF SMALL BOXES (REF. 17)

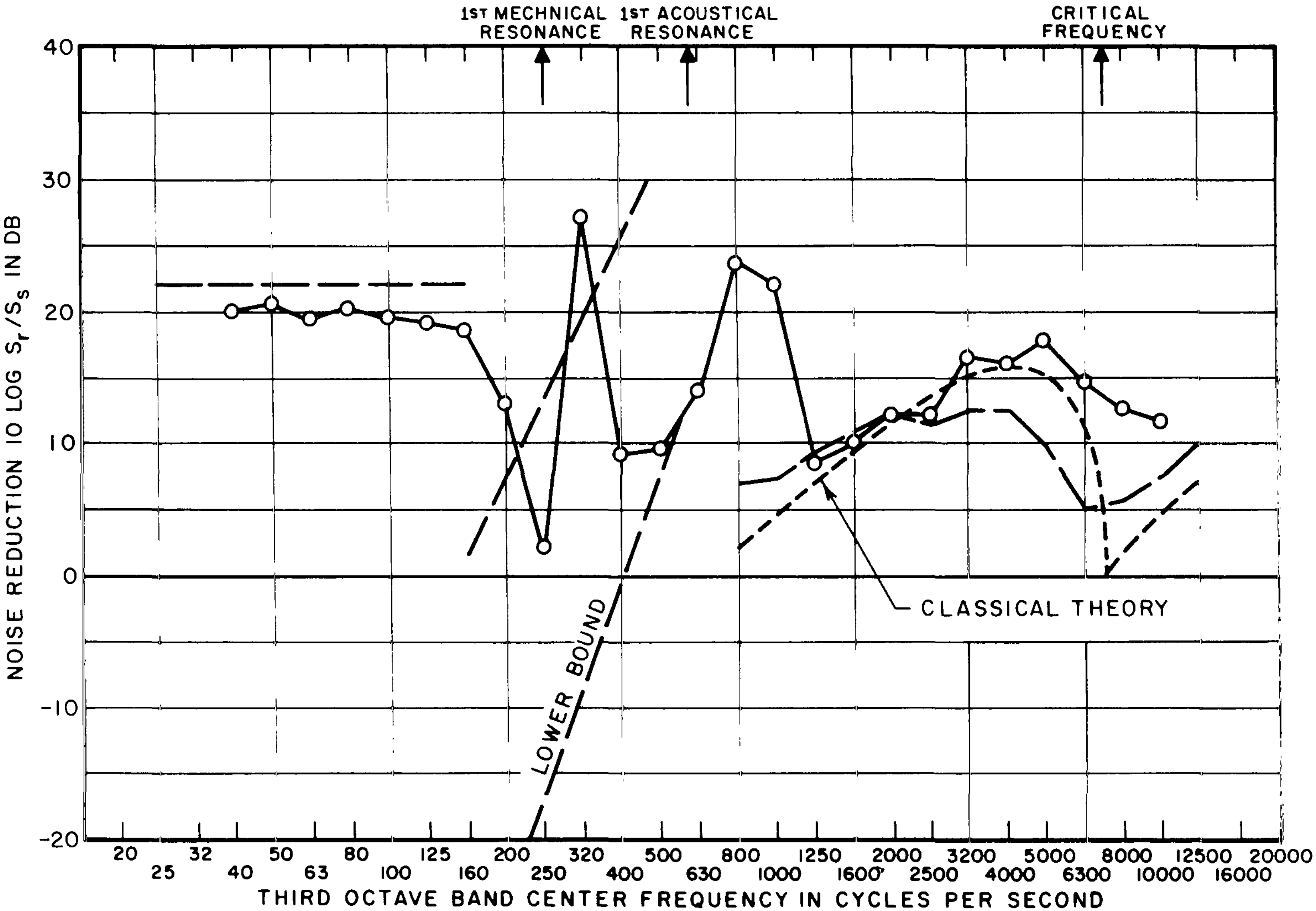

FIG. 19 MEASURED NOISE REDUCTION OF A SMALL BOX (REF. 17)

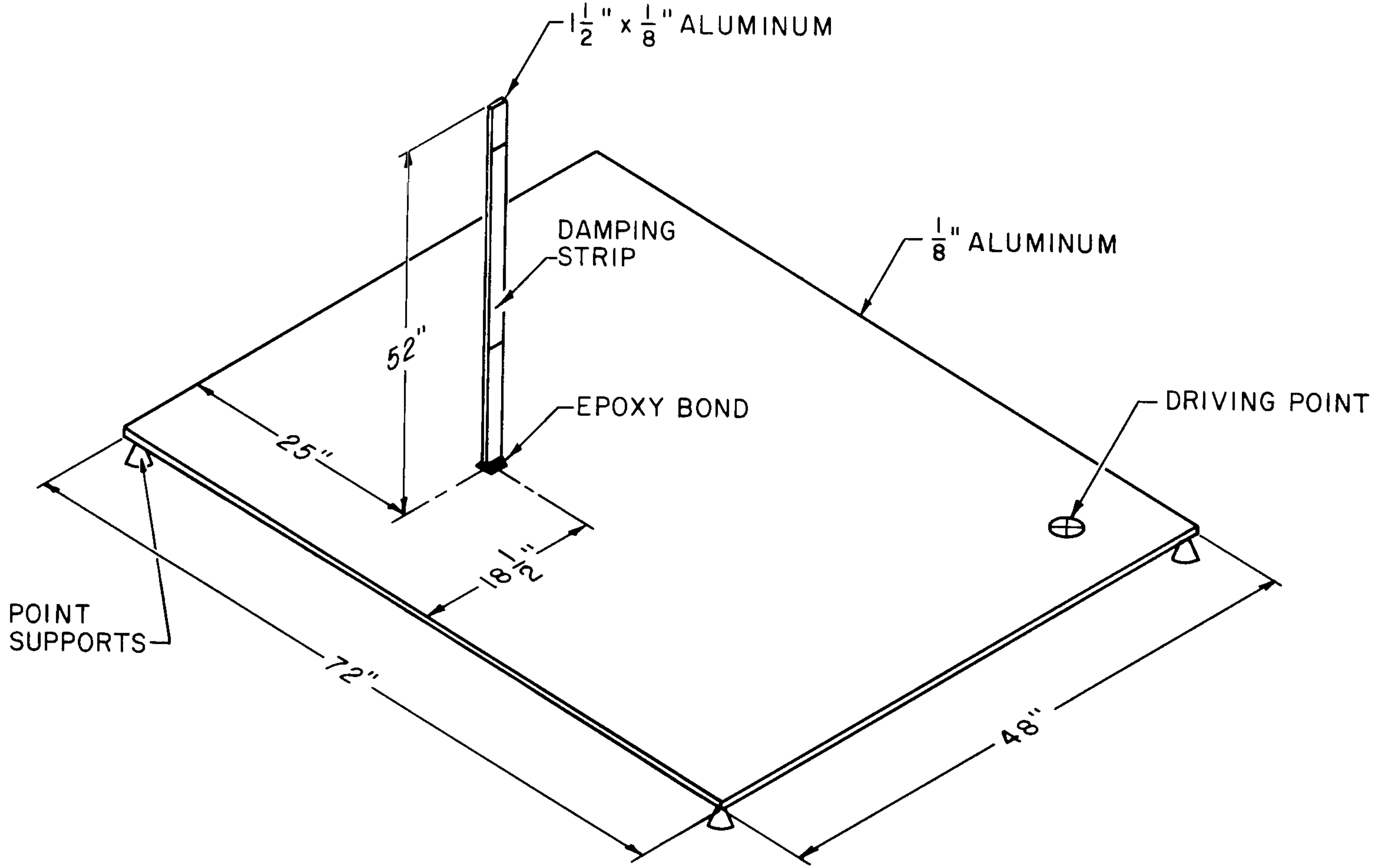

FIG. 20 SET-UP FOR BEAM-PLATE ENERGY SHARING STUDIES (REF. 19)

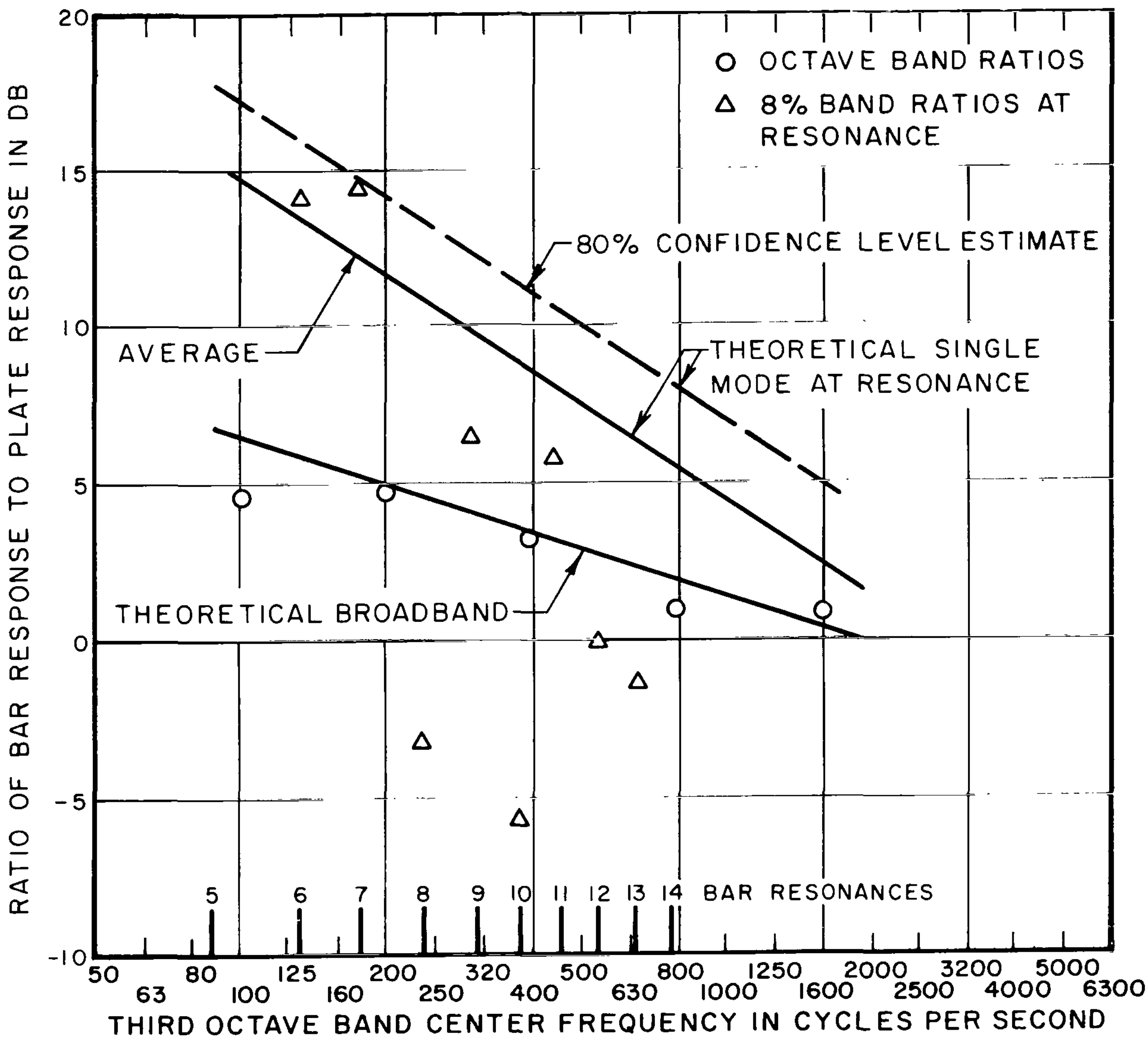

FIG. 21 RESPONSE RATIOS FOR BEAM CANTILEVERED TO A PLATE (REF. 19)

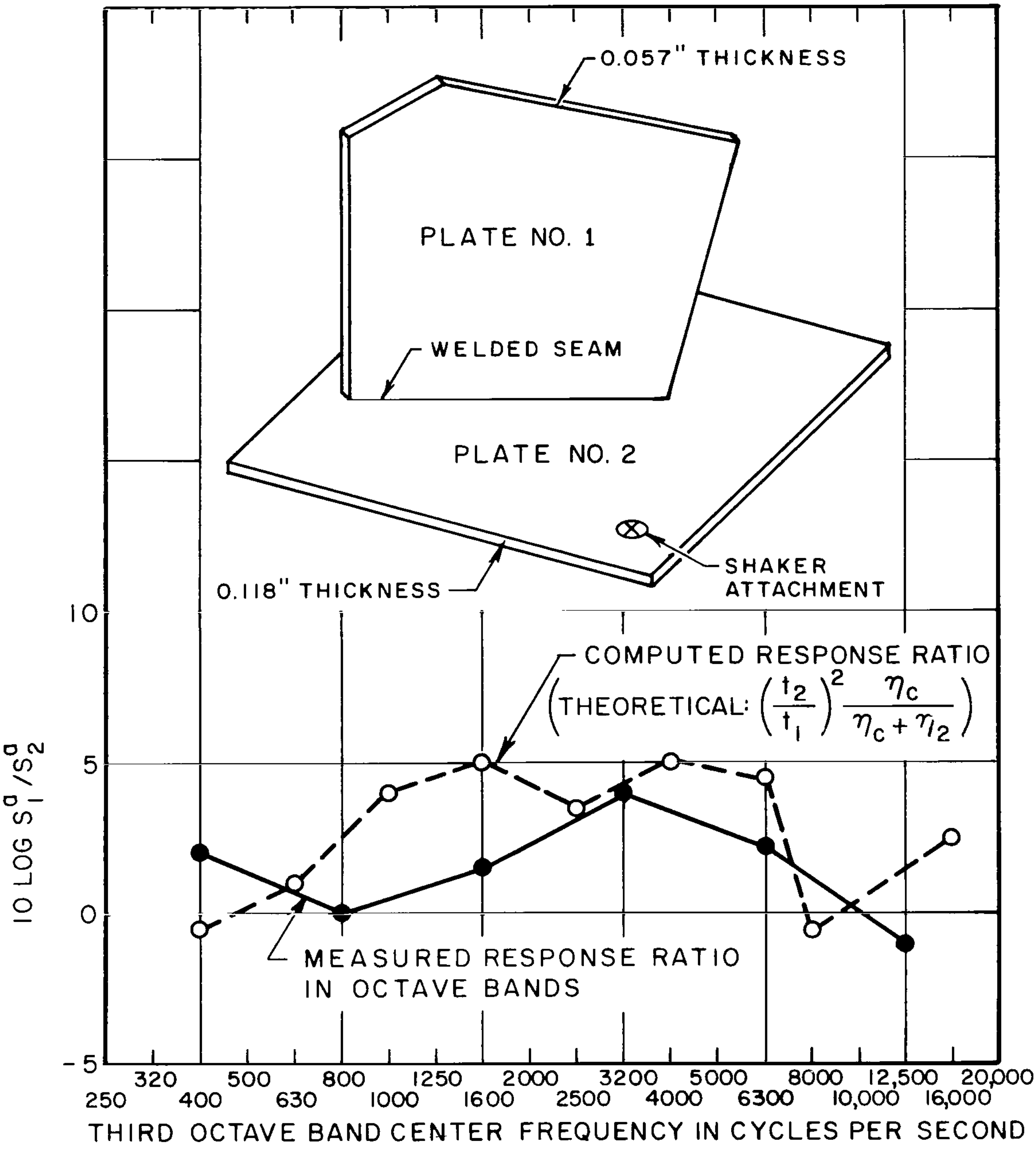

FIG.22 RELATIVE VIBRATION LEVELS OF CONNECTED PLATES, η_c = COUPLING LOSS FACTOR BETWEEN PLATE 1 AND 2, η_2 = INTERNAL LOSS FACTOR OF PLATE 2 (REF. 19)

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